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Multiplication with the Factor One, a Rare Mathematic Tool for Simplification and Unrevised DIN-ISO-ASTM-14577

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Abstract

The search for mechanical properties of materials reached a highly acclaimed level, when indentations could be analysed on the basis of elastic theory for hardness and elastic modulus. The mathematical formulas proved to be very complicated, and various trials were published between the 1900s and 2000s. The development of indentation instruments and the wish to make the application in numerous steps easier, led in 1992 to trials with iterations by using relative values instead of absolute ones. Excessive iterations of computers with 3 + 8 free parameters of the loading and unloading curves became possible and were implemented into the instruments and worldwide standards. The physical formula for hardness was falsely defined as force over area. For the conical, pyramidal, and spherical indenters, one simply took the projected area for the calculation of the indentation depth from the projected area, adjusted it later by the iterations with respect to fused quartz or aluminium as standard materials, and called it "contact height". Continuously measured indentation loading curves were formulated as loading force over depth square. The unloading curves after release of the indenter used the initial steepness of the pressure relief for the calculation of what was (and is) incorrectly called "Young's modulus". But it is not unidirectional. And for the spherical indentations' loading curve, they defined the indentation force over depth finally raised to 3/2 (but without R/h correction). They till now (2025) violate the energy law, because they use all applied force for the indenter depth and ignore the obvious sidewise force upon indentation (cf. e.g. the wood cleaving). The various refinements led to more and more complicated formulas that could not be reasonably calculated with them. One decided to use 3 + 8 free-parameter iterations for fitting to the (poor) standards of fused quartz or aluminium. The mechanical values of these were considered to be "true". This is till now the worldwide standard of DIN-ISO-ASTM-14577, avoiding overcomplicated formulas with their complexity. Some of these are shown in the Introduction Section. By doing so, one avoided the understanding of indentation results on a physical basis. However, we open a simple way to obtain absolute values (though still on the blackbox instrument's unsuitable force calibration). We do not iterate but calculate algebraically on the basis of the correct, physically deduced exponent of the loading force parabolas with $h^{3/2}$ instead of false "h^2" (for the spherical indentation, there is a calotte-radius over depth correction), and we reveal the physical errors taken up in the official worldwide "14577-Standard". Importantly, we reveal the hitherto fully overlooked phase transitions under load that are *not detectable with the false exponent*. Phase-transition twinning is even present and falsifies the iteration standards. Instead of elasticity theory, we use the well-defined geometry of these indentations. By doing so, we reach simple algebraically calculable formulas and find the physical indentation hardness of materials with their onset depth, onset force and energy, as well as their phase-transition energy (temperature dependent also its activation energy). The most important phase transitions are our absolute algebraically calculated results. The now most easily obtained phase transitions under load are very dangerous because they produce polymorph interfaces between the changed and the unchanged material. It was found and published by high-enlargement microscopy (5000-fold) that these trouble spots are the sites for the development of stable, 1 to 2 µm long, micro-cracks (stable for months). If however, a force higher than the one of their formation occurs to them, these grow to catastrophic crash. That works equally with turbulences at the pickle fork of airliners. After the publication of these facts and after three fatal crashing had occurred in a short sequence, FAA (Federal Aviation Agency) reacted by rechecking all airplanes for such micro cracks. These were now found in a new fleet of airliners from where the three crashed ones came. These were previously overlooked. FAA became aware of that risk and grounded 290 (certainly all) of them, because the material of these did not have higher phase-transition onset and energy than other airplanes with better material. They did so despite the 14577-Standard that does not find (and thus formally forbids) phase transitions under indenter load with the false exponent on the indentation parabola. However, this "Standard" will, despite the present author's well-founded petition, not be corrected for the next 5 years.

Keywords

Instrumental Indentation, One-Point, Spherical, Arithmetic Formulas, Reformulation, Factor One, Twinning Standards, Zerodur, Undue Fittings, Erroneous Standards, DIN-ISO-ASTM-14577 Revision, Petition, Energy-Law-Violation, Faked Data

1. Introduction

According to DIN-ISO-ASTM-1577 Standard, the spherical indentation is described with reference to Hertz as proceeding according to F_N prop $h^{3/2}$, but the

Sneddon, Love, and Johnson equations do not take care of the very strong sphere radius over depth ratio, or they violate against the energy law and they require data "fitting" of the depth values, as even published in [1]. We do not number such a "fitting formula":

$$\delta - \delta_{\text{contact}} = a_0^2/R\left[1+\left(1-P/P_{\text{adh}}\right)^{1/2}/2\right]^{4/3} - 2a_0^2/3R\left[1+\left(1-P/P_{\text{adh}}\right)^{1/2}/2\right]^{1/3}$$

The δ in this equation is a not allowable data-manipulation equation, which means penetration depth, and P means normal load. The fitted parameters are a_0 and P_{adh}.

The experimental data with a sphere are so manipulated and falsified that they counterfeit the use of the even false Johnson equation "$F_N = (4/3)h^{3/2}R^{1/2}E^*$", where E^* shall be the modulus. And this formula is a shortcut from the multi-terms formula with iterative processing, giving faked "F_N vs $h^{3/2}$ plots" of Sneddon: $F_N = \left[E/(1-n)\right]\left[\left(a^2+R^2\right)2h/a - aR\right]$ and from there a = cotg$b$) one obtains $F_N/(1-n)/E = 2h^2/\text{cotg}b - hR/\text{cotg}b + 2R^2\text{cotg}b$. The letters mean E: "Young's Modulus" (but that is faked unidirectional), n = Poisson's ratio, R = indentation radius a and b angles of the varying indentation triangle. K.L. Johnson made there from the equation $P = (4/3)\ h^{3/2}R^{1/2}E$ for spherical indentations, not taking care of the enormously varying R/h variation. The formula of Love used for spheres Hankel transforms and the theory of dual integral equations. He proposed the formula $F_N = E(1-n)^{-1}\left\{\left(a^2+R^2\right)2h/a - aR\right\}$, where $a^2 = R^2 - (R-h)^2$. When the substitution is performed, one obtains $F_N = E(1-n)^{-1}\left(2Rh-h^2\right)^{-1/2}(5R-h)h^{3/2}$ with a multitude of terms and with numerous different exponents on h. 20 years later Johnson came back by "summarizing Hertzian theory" with his above highlighted formula. It is not clear how he proceeded with the "summarizing". It is nevertheless clear that it is faked and totally in error, because it does not take into account the very strong variation of the missing R/h-ratio. For example, this missing factor varies from 1500 to 21 when R = 269 nm for a penetration length of 12.6 nm, or from 480 to 53.33 for a penetration length of 3.6 µm. It is clear that one cannot reasonably work without an R/h correction. The Loading Curve of Spherical Indentations in corresponding textbooks is so complicated that we renounce specific citations of it (cf. https://doi.org/10.4236/ampc.2023.136008).

It appears that more published spherical indentations use this data manipulation than correct spherical indentation reports. They have been challenged in Adv. Mater. Phys. and Chem.; DOI:10.436/ampc.200.1010016. It turned out, that the published spherical indentations of GaAs, Al, Si. SiC, MgO, steel, nickel superalloy, and PDMS (polydimethylsiloxane) were fitted to the faked Johnson's formula again without inclusion of the always very huge R/h ratio factor at the common depths. The most important dangerous errors from the necessary data manipulations are the disastrous errors in the DIN = ISO = ASTM-14577 worldwide Standard that will be hard to correct, as its Jurors are dominated by industrial voters against scientific evidence with "we did it so all the times", and not removing energy-law violating, and refusing easy detection of phase-transition with their onset force, and transition energy, even though after the fatal crashes of three airliner in short sequence FAA grounded six months after the final appearance of [2], which was blocked due to the present false 14577 Standard. It images corresponding microscopic images of stable micro cracks as nucleation sites including catastrophic breakage due to a new cracking mechanism. FAA grounded 290 of that type exhibiting previously overlooked dangerous micro-cracks at polymorph interfaces that are efficient nucleation sites for catastrophic crashes at higher forces than the ones for their previous formation. This was a clear indication that ASTM-14577 requires revision from its scientifically incorrect exponents on h, which must be corrected so that phase-transitions can be detected. That is also important for conical and pyramidal indentations where the false "h^2" instead of correct $h^{3/2}$ is used, while that error even denies the occurrence of phase-transitions under mechanical load, as it can never be detected with the false exponent. That proves to be disastrous. A chemical identical solid material can occur in different physical states. These are called phase-transferred, or twinned, and they are *modifications*, that are the same as *different phases*, or as *polymorphs*. They are physical different, thermally stable or unstable. Some are ambient stable, such as different minerals, while others are only stable under existing pressure and that case is mostly known from indentations (a different story is dangerous stable micro cracks within their interfaces, as long as higher forces are avoided). These are chemically identical but with different physical properties. These include before and after their phase-transition at characteristic forces and indentation depths different qualities such as hardness (manifested as different slopes of the F_N vs $h^{3/2}$ plot, so that a characteristic sharp kink ensues in an extremely precise way by equalization of the regression lines (Excel regression with error calculation for all data pairs or equidistant ones from published loading curves). That will be extremely useful for the now available on-site *in-situ* spectroscopic crystallographic analyses of these polymorphs: One chooses the end position for the one-kink cases. Correspondingly, for the two-kink cases, the best positions are shortly under the second kink and so on in multi-kink cases and always at the very end (for example, 5 new polymorph structures would be elucidated with the new *in-situ* X-ray or Raman technique as available from Bruker Hysitron, which opens new horizons for crystallographers. These facilities will be purchased and will open a new field of research so that both partners will profit. For example, when sodium chloride is again instrumentally indented up to 50 N loads (compare [2]). We repeat here the most well-known of the stable minerals calcite and aragonite that are both $CaCO_3$ and occur in natural mines, and can be mined by miners. All physical qualities, including phase-transition onset, force and kink energy and phase transition energy, are different. They are also different under their different crystal faces (their comparatively low values point to twinnings under [3]). Further well-known technically used different modifications = polymorphs = phases are for iron and steel, but the overall list is huge and extremely important for industrial use of materials for phase-transition detection under load by the reliable, easy, cheap, and fast technique. No other technique affords that possibility, and we have high precision due to regression analyses. One just has to accept the scientific truth ($h^{3/2}$ instead of false "h^2" for the normal force of cones and pyramids, or $h^{3/2}\pi(R/h - 1/3)$ for the force of spheres.

2. Methods

All values are algebraically calculated with equidistant force-vs depth data pairs (at least 20 from published loading curves, or digitizer (http://www.softpedia.com), or all thousands of initial F_N/h date pairs) with Excel regression and print. The F_N/h data are directly inserted into the corresponding data columns, which are combined with the necessary factor columns for the common least square sum regression calculation routine. The print allows for choosing the range before and after the kink point together with the confidence values that are for thousands of data pairs always larger than "four nines". We therefore obtain the depth and applied force of it very precisely. The identification of these branch equations provides the values for the phase-transition onset force and depth$^{3/2}$. Using these (and the F_a value for correction of initial effects) allows for the calculation of the phase-transition energy, according to the corresponding formulas in [3] or [4]: The calculation tools for the equations of the straight line branches that form the characteristic kink positions are used in the case of phase-transition(s) of the slopes k (e.g., mN/μm$^{3/2}$), (which is the penetration resistance k = physical hardness) and axis cut (e.g., mN). The combination with the normal force (F_N) vs $h^{3/2}$prints of the linear plots with intersecting structural phase-transition (or twinning) kinks are then separately regressed with Excel error calculation, giving the different slopes from zero to F_{N1kink} (k_1[mN/μm$^{3/2}$]) and $h_{1kink}^{3/2}$ [μm$^{3/2}$] (different factors of 1000 are here, of course, possible), and the initial axis cut F_{a1} (mN) for the correction of any initial effects. In the case of only one kink the k_2, $h_2^{3/2}$ and F_{a2} are at the end of the scan. In the case of a second kink, the terms are correspondingly used for the second kink. The k-values are the penetration resistance, which is the physical hardness for indentation with specified indenter geometry. These values are used for the calculations of the applied energies W_1 and W_2 [mNμm] before and after the kink. Their sum is then subtracted from the total applied energy to a chosen end-force that is divided through the Δh from the kink to the chosen end, for obtaining the normalized phase-transition energy per μm. The positive values are from endothermic phase-transitions, whereas the less frequently found negative values are from extremely dangerous exothermic phase-transitions. The spherical indentations can be analysed in the same way as the pyramidal or conical ones, if the correction term $\pi(R/h - 1/3)$ is multiplied to every equidistant $F_N/h^{3/2}$ data-pair of the spherical experiment. Thus, the phase-transition onset has the same meaning, and the transition energy can be equally calculated in the same way, obeying the energy law, using the correct exponent, detecting phase-transitions, and calculating the phase-transition energy. For the spherical indentations, we plot F_N vs $h^{3/2}\pi(R/h - 1/3)$ and proceed accordingly. Experimental results are seen in [4].

The hardness values and the so-called "Young's moduli" (one should determine these with ultrasound or with Hooke's experiment) from the 14577-Standard are no physical values and they are not reproducible. The 3 + 8 free-parameter iterations do not help in making them reproducible. And that is also due to the twinning of their basic standards in the measurement routine with fused quartz or aluminium. The twinning onset and their structural transition remain undetected or uncorrected in both cases. And the twinning is facilitated by not controllable impurities in the ppm range of their concentration. We can compare the differences (errors) with the physical hardness and the nevertheless calculated conversion energies [3] or [4] per μm from their printed loading curves. The difference in the Hysitron Handbook on pages 57 and 59 for fused quartz is for k_1 (the slope of the F_N vs $h^{3/2}$ curve) is 8.43%, and for the twinning energy 9.97%. This becomes more drastic when compared with the Handbook of CSIRO-UMIS. Now the corresponding deviation of the twinning onset k_1-value amounts to 28.39 %. This shows that not only is the violation of the energy law a violation, but the instrument's calibration standard also does not produce repeatable values worldwide. They are dangerously useless, when different materials must be compared. The suggested high-tech Zerodur as a certified standard for the Revision of the 14577 Standard was not accepted.

3. Results and Discussion

Due to the importance of the subject, we repeat here the deduction of the physically and mathematically correct deduction of the load-depth formula of spherical indentations [4]. One produces both an indentation volume and, in addition to that, pressure from some plasticity (that is often termed "total pressure" p_{total}, also requiring force and thus energy). For the indentation volume from spheres, we need the volume of the sphere-calotte. It is tabulated and equally found on the internet as the two-member Equation (1). Both members contain the depths h, one with exponent 2 and one with exponent 3 (After an out-multiplication). That makes its use complicated, when compared with the not numbered false formulas in the Introduction, except the false one-member equation of Johnson that therefore found acceptance in DIN-ISO-ASTM-14577 and so much appreciation that about half of the published spherical indentations manipulated their experimental data, so that these were fitted for "agreeing" to it, again with the violation of the energy-law by the followed ISO-hardness and so-called iterated "Young's modulus", despite its not being unidirectional. In this uncredible false situation, we looked for a situation to create in a mathematically correct way for reformulating (1) into a one-member equation and developed the title solution that is simply multiplication in this case with $h/h = 1$, to give Equation (2). In that very elegant way, we obtain both a correct one-member equation for the sphere-calotte volume that also contains the dimensionless number R/h ratio that can be added to −1/3 in the parentheses of (2). However, this strongly variable number in the parentheses of (2) has to be added to every data pair for volume with respect to the calotte radius.

$$V_{\text{sphere-calotte}} = \pi h^2 \left(R - h/3 \right) \quad (1)$$

(1) gives by multiplication with 1 = h/h, which stays thus identical (2)

$$V_{\text{sphere-calotte}} = \pi h^3 \left(R/h - 1/3 \right) \quad (2)$$

We used the rewritten Calotte volume Equation (2) and used the multiplication with 1 (here 1 = h/h). The formula (2) is a correct form of the tabulated calotte volume and our procedure could certainly have helped to simplify the unnumbered complicated formulas in the Introduction, but we renounce using it here, because their basis (using E (elastic modulus) and requiring energy violation instead of the indenter geometry) is not worth-while. Nevertheless, multiplication with a factor of one might be correspondingly useful in further scientific cases for simplifying complicated formulas. By here using (2), we obtain with πh^3 out of the parentheses only one variable now with h^3 and within the parentheses, we have only two dimensionless numbers R/h that are subtracted by 1/3. That is much better than the out-multiplication, which gives two terms, one containing h^2 and the other containing h^3. And we now have the strongly varying correction term R/h for the dimensionless correction factor that complicates spherical indentations, but it is easily treated by hand or computer. These correctly analysed spherical indentations are more related to the much more complicated and expensive diamond anvil pressurizing experiments, while conical or pyramidal indentations are not so [5]. The tip radius must be precisely detected with non-touching AFM, but not with the unfortunately still common ISO-ASTM technique, which is by far too imprecise, as the calotte volume changes extremely with every data pair at every impression depth separately, for a plot of F_N versus variable $\pi h^{3/2}(R/h - 1/3)$ (2). The total force and pressure are proportional to V_{calotte} (1). As all indentations produce both volume and pressure, we have to take care of both effects and must not follow the DIN-ISO-ASTM-14577 Standard that uses all applied force for the volume formation falsely so that the actual remaining pressure and eventually produced *plasticizing are presumed to be formed with zero energy, which is a strongly forbidden claim*. The present author continues to tell the worldwide DIN-ISO-ASTM-4577 that such behaviour violates the energy law even at a recent revision session of DIN (that Agency formulates these Standards), but the Jurors are dominated by industrial voters clearly against scientific evidence and still (on 11.11. 2024) voted (that is shameful and frequently dichotomised [6]) for still violating the energy law, by not changing the false exponents for the easy detection of phase-transitions that could have prevented three airliners' fatal crashing. Dichotomy is here knowing that their experimental loading curves do follow $h^{3/2}$ but nevertheless using "h^2" for their calculations, iterations, simulations, or sometimes even for the manipulation of their data, so that these seem to follow "h^2" (see such a not numbered equation in the Introduction Section). That has been disclosed and published in several cases. And by knowing that after the final appearance of [2] (containing acknowledgement for the measurement at the Company's high force instrument), which also contains the photographs of the micro-cracks at the polymorph interphases due to phase-transition under load from a VH-Z500 digital microscope, as purchased of the present author. The FAA grounded 290 airliners of the same type as rapidly as possible, that is 6 months after the final appearance of [2] for 18 months. Clearly, that used an extra check to cover all active airplanes, as legally enforced. These micro-cracks nucleated the catastrophic crashing at a higher force than before their creation. And that is also imaged in [2] for the model material, together with the warning from the new crashing mechanism, all in the paper [2]. All of the crashes of 3 airliners in short sequence (over China, the Indian Sea, and Ethiopia) had apparently such previously overseen micro-cracks (the newspapers wrote only "cracks", but only micro-cracks are hardly seen) that were overseen by their legal 6 months' checking. Clearly, FAA enforced it exactly 6 months after they became knowledgeable of the content of my open-access paper with the NaCl model [2] right after its final appearance (after four years of delay by rejections of Journals in Germany, USA, and Switzerland. It was always due to their Peer Reviewers that cited DIN-ISO-ASTM-14577 and convinced the journal Editors to use it. None of these cared for the errors in that "Standard". So, one may ask: who is responsible? And probably all were already Dichotomists [6] and knew it better at that time. However, they were biased and defended Standard 14577 despite its more than easily seen physical errors. But they had taught it before to their students and had so published their dichotomous claims. The three fatal airliner crashes could clearly have been avoided with revised Standards, by no longer forbidding phase-transitions. FAA now immediately required its additional 6 months recheck of all active airplanes by also looking for micro cracks (only 1 or 2 μm long) by covering the safety-checking of all active airplanes. The result was the at once enforced grounding of 290 airliners of the same type as the crashed ones for 18 months. It was due to such micro cracks, as everybody could read in the world-wide newspapers (I just added the "micro", because every safety check would have objected to clearly visible cracks). ***FAA so certified that phase transition forming the interface with ground material is at risk by therein formed micro crack formations that grow at higher mechanical load for the catastrophic fatal crashing, here of the three airliners in short sequence.*** These worldwide beneficial groundings prove not only the acceptance and agreement of FAA with the dangerous phase-transition under indentation with their turbulence's loadings, but it should immediately change the technical agreement of DIN-ISO-ASTM-14577 Standard (all of it is the German DIN-14577 Standard) of such dangerous phase-transitions under load. It appears important enough as the total costs next to the deplorable more than 300 dead amount to 100 billion Dollar admitted costs of the Vendor and Constructor of its whole modern airliner fleet. It is so easy to check a new material for the pickle forks that connect the wings to the fuselage, which might have different advantages over older material. However, more important is the comparison of their phase-transition data. One must only perform instrumental indentation and physical/mathematical correct analyses for phase-transition onset and force, and phase-transition energy for comparison of related previous materials. The present author apart from his publications also lectured in USA at the respective scientific meetings, where he discussed with people from airplane and space flying industry. There was enough knowledge around the subject. And there is mankind that wants safety and no risks when flying new airliners that were not controlled for comparably poor phase-transition values. Those highly qualified people must be open to scientific checks of Standards that are developed by industrial Jurors in Germany. At least these Standards deny responsibility for physical correctness. The errors in the standards are so obvious and easily seen. We at least now hope that ASTM will refuse to accept any uncorrected DIN-ISO-14577 Standards or require their thorough scientific revision, which at present still failed despite my scientific petition due to overwhelming majority votes of industry owners or managers who are interested in retaining their standards, rather than correcting them (for example, it means that one allows for dichotomy with the exponents, denies all phase transitions under load, retains twinning standard materials, uses 3 + 8 free parameter iterations, violates the energy law, renounces of business with phase-transitions, and further applications). All of that can be easily found and calculated by the indentation with cone or pyramid or sphere. And despite pointwise correction factor for sphere, it is much easier than the anvil pressurizing to find phase transitions, but these are without their onset and energetic terms. Only the indentation proves the (indenter related) onset forces and transition energies. It is hoped that the colleagues of DIN-ISO-14577 and in the USA at ASTM will not accept such voting of DIN in Germany. The Industrials' "reasons" will for example be: "such changing of DIN-14577 would be too extensive and they know the practice better than a correcting Scientist" (that is correct but reverse for the data calculation), and it is clear: "Phase-transition into polymorphs with changed mechanic properties will be the most important part of instrumental indentation. It must become a correct part of the 14577 Standard". All Jurors, except the present Author and the neutral Leader of a scheduled session, denied the petition for urgent correction with such highly profitable chances. An uncredible dangerous majority against the scientist's petition is surprising, even though most of the available academic teachers are still using false exponents in a situation of dichotomy [6]. They know that they use the false exponent and avoid profiting from the correct one's completely new applications. That is particularly true for the instrument sellers and constructors, who apparently refuse the undeniable widespread new possibilities of indentation without seeing the new success in their business in selling new instruments and new accessories for the old ones. If phase transitions are longer blocked by DIN-ISO-ASTM, new research fields must wait to be opened. Thus, instrument Builder-Seller also loose an important part of their possible business.

We now repeat shortly how we got to our criticism. For the separation of applied normal force into volume formation F_{NV} and total pressure formation $F_{Nptotal}$ we use Equation (3) and determine the exponent's m and n. Their separation corresponds to the one for the conical or pyramidal indentation and for the spherical indentation, as in the total pressure. And also the total normal force is proportional to h^3 (4). When $n = 1/3$ then $m = 2/3$, because these exponents must add to 1 and one obtains (5).

With Equation (2) we obtain (3). This reveals that p_{total} and thus also $F_{Nptotal}$ are proportional to h^3 of the immersed sphere, which occurs with the same proportionality. And (3) reveals that $F_{Nptotal}^{1/3}$ and p_{total} are prop h^3, and that $F_{Nptotal}^{1/3}$ is prop to h.

For not violating the energy law, we have to distinguish the force and energy for the penetration (volume formation V) and for the total pressure formation residual p plus eventual plasticity from p, that is p_{total} (3). And we will have to determine the exponents m and n, the sum of which must be 1. With p_{total} and also with $p \propto h^3$ in (4) we obtain also $F_{Nptotal}$ and equally F_{Np}. We obtain so (4), as F_{Np} does not contribute to the depth. Clearly, the $F_{Np}^{1/3}$ proportionality with h gives the exponent $n = 1/3$ on h for pressure and thus $m = 2/3$ for volume in (3).

$$F_N = F_{NV}^m F_{Nptotal}^n \tag{3}$$

$$P_{total} = KV\,;\; V_{Sphere\text{-}calotte} = \pi h^3 \left(R/h - 1/3\right), \text{ according to (2)} \tag{4}$$

$$\text{Equation (4) reveals: } P_{total} \text{ and also } F_{Np} \propto h^3 \tag{5}$$

Formula (5) reveals the $F_{Np}^{1/3}$ proportionality to the depth h, but $F_{Np}^{1/3}$ to the depth, and

$$\text{Also } F_{Np} = \pi h_{ptotal}^3 \left(R/h - 1/3\right) \tag{6}$$

As (5) shows $V_{Np} \propto h_p^3$, and as $(R/h - 1/3)$ is a dimensionless control factor for every single data-pair) so that the h in the parentheses of (4) has lost its dimension, as denominator of the indentation radius of h. And it does not contribute to the depth. Nevertheless, when $n = 1/3$, m must be 2/3 according to Equation (1), and this gives Equation (7).

$$F_N = F_{NV}^{2/3} F_{Nptotal}^{1/3} \text{ or } F_{NV} = h^{3/2}\pi\left(R/h - 1/3\right) \tag{7}$$

The exponent 3/2 on h in Equation (7) reveals that while the instrumental indentation is $F_{Nsphere\text{-}calotte}$ vs $h^{3/2}\pi(R/h - 1/3)$, only the fraction $F_N^{2/3}$ is responsible for the penetration and its depth is $h^{3/2}$. This is expressed with the searched-for Equation (7), where we no longer need the index $_V$. The unavoidable pressure/plasticizing factor $F_{Np}^{1/3}$ is lost for the depth. This is the physical reason for the exponent 3/2 on h instead of recently assumed 2 for cones and pyramids. The successful applications of (7) for experimental spherical indentations and also for the exclusion of various manipulated ones (see, e.g., the not numbered fitting Equation in the Introduction) that are analyzed or recognized and disregarded are in [4] and [5].

4. Conclusions

Instrumental nano-, micro- and macro-indentations are most important for the detection of phase transitions under load. They appear reliably in any loading curve at a well-defined load, sometimes with rather low force or at other materials only with high and very high forces. And there are also consecutive phase-transition kinks in many cases. The crystallographic structural characterization is facilitated by on-site *in-situ* spectroscopy, for example, with focussed X-ray analysis. The maximum concentration of the transformed material (at the end or close to the second kink) can be profitably chosen. All of that is material specific and can only be qualified with their indentation depth and phase-transition energy in view of onset-energy and phase-transition energy. They depend significantly on the indenter tip geometry. All instrumental indentation performances (except the use of twinning force standards for iterations) are very precise, because DIN-ISO-ASTM gives good advice for how the experiments are reliably performed, so that really experimental published force-depth curves can be analysed and differentiated from not (completely) experimental ones. However, these produce not physical values but only relative non-physical phantoms that are never comparable with absolute values. The instrumental indentations are technically well developed and much easier and cheaper available than any other technique for the phase transition detection (even though some authors claimed to have used the present 14577 Standard exponent (that is the false "h^2" one), but their printed parabola reveals actually the correct linearity with $h^{3/2}$. They do so to please the Peer Reviewers, who cannot see it without an exponent check. We identified such cases even in the Handbooks of the Instrument Sellers in [6] and call it dichotomy. The formed polymorphs that are only revealed with our correct $h^{3/2}$ analysis have different mechanical properties (be it only twinning or structural) and the different polymorphs form dangerous interfaces between the polymorphs with the increased risk of micro-cracks (only 1 or 2 μm long and stable, except crash at higher force). The high-resolution microscopy with no further stress on the NaCl sample at ambient conditions in a closed vessel for more than half a year showed it. But they are nucleation sites for catastrophic crashing at higher forces than those at their production. Thus, all technical materials must only be stressed below the formation forces for the first newly created polymorph. Viable comparisons of materials with the same indenter geometry and force standard require that onset and endothermic transition energy are as high as possible.

Importantly, the loading curves must be analyzed with the correct normal force versus $h^{3/2}$ relation (cones and pyramids) or $h^{3/2}\pi(R/h - 1/3)$ (spheres). The incorrect analyses against "h^2" of the DIN-ISO-ASTM standards never show any phase transitions: The most important application for easily obtained and, for the first time, very precisely qualified materials with respect to onset force and comparative force by regression is only possible by analysis using the correct F_N vs $h^{3/2}$ relation. That is not possible with the incorrect DIN-ISO-ASTM-14577 "h^2" for cones and pyramids (or without the R/h term for spheres). Similarly, the energy of phase-transition by regression analysis (Excel) values for the arithmetic calculation of the phase-transition energy is only possible with the correct formulas. That is the easiest and for the first time possible characterization of these important values. They must be the larger, the better (when endothermic) and one can select the most advisable material for technical safety, because dangerous phase transitions can only be avoided by keeping mechanical stress always well below the onset force. For example, the pickle fork material of airliners must withstand all turbulences. Thus, the search for better new material is highly relevant for safety reasons. When instrumental indentation is not executed, or if such loading parabolas use the present phantom values of the DIN-ISO-ASTM-14577 Standard (e.g., by using "h^2" instead of $h^{3/2}$), one risks catastrophic crashes at stronger turbulences than the previous ones. It must be repeated here: Three airliner crashes in a short sequence (over China, Indian Sea, and Ethiopia) unfortunately, with all crew and passengers dead, it took the open access publications' [2] final appearance that FAA understood and immediately ordered the rechecking of all airplanes for hitherto not complained micro (1 - 2 μm long) cracks within the extra obligatory 6 months safety check of all active airplanes. And all of a sudden, it grounded 290 airliners of the same crashed type for 18 months, due to such micro cracks. It is hard to understand that my petition at DIN (that formulates the Standard 14577 for 5 years) failed in a majority vote. Thus, DIN did not immediately change the exponent on h (and did not use the new formula with R/h corrections along the initial sphere calotte). That continues to prevent the check for comparative phase-transition onset values. But the same written arguments of the petition have not changed and contain examples for tired-out bridges and other buildings, windmills, etc., which are a present problem almost everywhere. The risk for DIN-ISO is that ASTM colleagues might not agree (as the FAA groundings of 290 airliners due to phase-transition now found the reason by micro-cracks). The damage, in addition to the deplorable deaths of all Crew and Passengers, was at least 100-billion-dollar financial costs for the Constructor and Seller of its complete modern fleet. Not to forget uncountable further, less drastic events that are in their hand. It contains my accusation that the deplorable fatal crash could have been avoided by a correction of their dangerously false Standard 14577. It is outlined that their Standard blocked the paper [2] to appear well before these crashes for several years, because all Peer Reviewers relied on Standard 14577 and so convinced also the Editors. The paper [2] could have been published years before the crashes, so that FAA would have learned and acted long before. Such a new cracking mechanism was imaged and warned from it. But unfortunately that was delayed by repeated blocking for years of anonymous Peer Reviewers in Germany, USA, and Switzerland, every time with very long time for rejection, then rebuttal, then again final rejection by the Editor to make only the paper free for new submission in China. Over and over the same story with reference to the false DIN-ISO-ASTM-14577 Standards that use the false exponent on h, or do not request the R/h ratio (for spheres), allow for manipulated experimental data to make them follow "h^2", and use simulation instead of algebraic calculation, or iteration with the infernal number of 3 + 8 free parameters for fitting to totally unsuitable also for its twinning standards fused quartz or aluminium. These are unsuitable due to their uncontrollable twinning, depending on the uncontrollable ppm-range concentrations of impurities. Nevertheless, they refuse to use an absolute force standard without twinning, like Zerodur, which is a worldwide super material used for the highest precision astronomic mirrors, space flying materials, and hot plates in kitchens worldwide. Only unbiased Reviewers and Editors from China looked at the highest essence of the paper [2], the highest need for scientific and practical value, and the most important safety reasons. But that was after many years of blocking by the false DIN-ISO-ASTM-Standard-14577. It needed the three catastrophic airline crashes that happened in short sequence, and only after the appearance of [2] could the FAA require the recheck of all airplanes and microcracks at polymorph interfaces. The reason became clear in my clarifying paper [2] that had a too-long wait for open-access publishing. Again, the crashes could have been avoided had Standard 14577 not been used with its false exponent 2, which actually forbade the search for phase transitions under load.

And even worse, I dare now to call it "shameful", that the violation of the energy law since 1972 was tolerated, even though the present author complained to lecturers and published in 2013 [7] how easy it is to quantify the amount of work that shall be produced with zero energy (20% for a parabola with exponent 3/2 and 33 % for a parabola with exponent 2). And that was long before complained about in numerous worldwide lectures without wide enough appreciation. What shall the physics teacher tell his pupils of such capital error, as is tolerated (except by the present Author) for up to more than 30 years in all relevant courses/instructions of universities? The violation of the same type was tolerated within the worldwide acclaimed technical standards! The teacher's responsive teaching object might be the common wood-cutting that is clearly related to indentation. He will certainly never insist that all energy from the hatched is for its penetrating into the wood, because the horizontal separation into parts also requires energy. In the math course, they will hopefully also discuss the influence of the hatched angle with sharp angles and others with obtuse angles. It is certainly an interesting practical exercise for the solution in the math course for the practical application of simple trigonometry.

In 2016 [8], it was shown how easy it is to disprove the exponent 2 on h for conical or pyramidal indentations of [9] and DIN-ISO-ASTM-14577 by an undisputable strict deduction of $h^{3/2}$ for conical and pyramidal indentations. And the drastic violation of the energy law is also complained about in [7]. But it is still tolerated in the present Standard 14577.

Nevertheless, my direct petition failed against all the scientific arguments against the majority votes from the industrial voters. They were apparently afraid of the immense changes of their enterprises as long as they could rely on the long-used "Standards" of DIN-ISO-ASTM-14577 against all of the scientific objections and against FAA, as long as they could still refer to the acclamation of a prevailing self-inflicted dichotomist's scientific crowd, which uses their purchased instruments with their black-box installations. Most of the addressed Peer Reviewers grew up with it. A hopeful sign might be the interest of the technical DIN discussion Leader, who expressed interest but did not vote. The admission of the relevant industry requires obeying DIN-ISO-ASTM-14577. However, the texts of these standards deny expressively any legal liability for correctness. That means everybody must use the prescriptions at their own risk. And what about any private liability? Shall any final liability be transferred to the involved Individual? Industry Owners have to sign the admission document for their factory, containing Standard 14577, even if they are not scientists who can easily judge possible errors of Standard-14577. These are open legal questions. However, one can almost be certain that in the future, the relevant industries will use the search for phase-transition onset and conversion energy comparison of materials. They only need indentation, Excel calculation of the F_N vs $h^{3/2}$ calculation for the phase-transition onset values (force, depth, and axis cut) and then calculation of the conversion energies with the provided formulas: [2] through [4]. As they do it for safety reasons, they can claim that this was done in addition to what they signed for admission. They must avoid similar very big disasters by choosing their best possible material, and FAA should be their Witness. Such behaviour would not only be helpful for airliners' pickle forks, or every other site (e.g., cables under stress, turbine blades, etc.). All mankind would profit from more safety with technical objects in daily life. Physical laws and calculation rules are helpful. Good Luck!

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Ebenstein, D.M. and Wahl, K.J. (2006) A Comparison of JKR-Based Methods to Analyze Quasi-Static and Dynamic Indentation Force Curves. *Journal of Colloid and Interface Science*, **298**, 652-662. https://doi.org/10.1016/j.jcis.2005.12.062

[2] Kaupp, G. (2018) Six Polymorphs of Sodium Chloride upon Depth-Sensing Macroindentation with Unusual Long-Range Cracks Requiring 30 N Load. *Journal of Material Science & Engineering*, **7**, 473-483. https://doi.org/10.4172/2169-0022.1000473

[3] Kaupp, G. (2019) Physical Nanoindentation: From Penetration Resistance to Phase-Transition Energies. *Advances in Materials Physics and Chemistry*, **9**, 103-122. https://doi.org/10.4236/ampc.2019.96009

[4] Kaupp, G. (2020) Valid Geometric Solutions for Indentations with Algebraic Calculations. *Advances in Pure Mathematics*, **10**, 322-336. https://doi.org/10.4236/apm.2020.105019

[5] Kaupp, G. (2020) Real and Fitted Spherical Indentations. *Advances in Materials Physics and Chemistry*, **10**, 207-229. https://doi.org/10.4236/ampc.2020.1010016

[6] Kaupp, G. (2024) A Way out of World-Wide Indentation Dichotomy in Materials' Science. *Modern Concepts in Material Science*, **6**, 2-5. https://doi.org/10.33552/mcms.2024.06.000634

[7] Kaupp, G. (2013) Penetration Resistance: A New Approach to the Energetics of Indentations. *Scanning*, **35**, 392-401. https://doi.org/10.1002/sca.21080

[8] Kaupp, G. (2015) The Physical Foundation of $F_N = kh^{3/2}$ for Conical/Pyramidal Indentation Loading Curves. *Scanning*, **38**, 177-179. https://doi.org/10.1002/sca.21223

[9] Oliver, W.C. and Pharr, G.M. (1992) An Improved Technique for Determining Hardness and Elastic Modulus Using Load and Displacement Sensing Indentation Experiments. *Journal of Materials Research*, **7**, 1564-1583. https://doi.org/10.1557/jmr.1992.1564